A generalized action principle for D=4 doubly supersymmetric membrane

Aleksey Yu. Nurmagambetov
NSC Kharkov Institute of Physics and Technology
Akademicheskaya 1
310108 Kharkov, Ukraine
e-mail: kfti@rocket.kharkov.ua

Abstract

In this review I discuss some recent results concerning D=4 doubly supersymmetric membranes within the framework of geometrical approach obtained in the collaboration with Igor Bandos, Dmitrij Sorokin and Dmitrij Volkov.

Starting its development from the beginning of the 70's the string theory has taken up a stable place into the modern theoretical physics and nowadays it can be considered as one of the most appropriate candidates for the construction of the Unified Theory. One of many at the moment, but not the only one, because if we have made a very important step from point to string why don't we go further towards a consideration of membranes and other extended objects? In the other words we need find an answer whether the string theory is an unique possibility for unification of all interactions or it is not so.

The first steps of membrane physics having in some sence purely academic value were more than unassuming in comparison with the impressive achievements of the (super)string theory. But discovering D=11 supergravity which is a low energy limit of D=11 *supermembranes* arroused an interest to the relativistic supermembranes as an alternative to the superstring theory.

The absence of Green – Schwarz superstring covariant quantization procedure and impossibility to construct a perturbative picture of string interactions were ones of many reasons which stimulated the investigations of the string/membrane duality [1] at the low – energy effective action level in the framework of non – perturbative approach. These investigations have legalized a status of higher – dimensional extended objects (super – p – branes) still more and incited the search of a version of super – p – branes description being suitable for covariant quantization.

In contrast to the superstrings the problem of super – p – brane covariant quantization is hampered first of all due to the more complicated structure of these objects, giving essentially nonlinear equations of motion, as well as by other obstacles, containing, particularly, an infinite reducible κ – like symmetry. Nowadays there are no recipes to solve completely membrane equations of motion even at a pure bosonic level, although the recent papers of J. Hoppe et al. [2] inspire a hope for a positive reply to this question. Unfortunately, there is also no idea how to quantize gauge theories in the presence of infinite reducible symmetries.

In spite of difficulties mentioned above, at least the κ – symmetry problem doesn't seem to be so dangerous now. The progress in solution for this problem is closely related to the development of twistor – like [3] and Lorentz – harmonic [4] approaches to the doubly supersymmetric particles and strings.

In the twistor – like approach the local fermionic κ – symmetry, generated by infinite reducible constraint, is replaced by local worldsheet supersymmetry which is irreducible by definition.

However, clear understanding of geometrical nature of the κ – symmetry in the framework of doubly supersymmetric twistor – like approach is not the whole story because the general structure of superparticle and superstring action has the following form (see for details [6]):

$$S = \int d^p\xi \, d^N\eta \, P_K \Pi^K + \quad generalization \;\; of \;\; WZ \;\; term \tag{1}$$

with P_K are the superfield Lagrange multipliers and superfield functions Π^K containing all the geometrical constraints of supersurface embedding into a target superspace and equations of motion. The geometrical and physical sence of superfield Lagrange multipliers is shadowed, moreover this theory possesses another infinite reducible symmetry [6] which eliminates auxiliary degrees of freedom contained in the superfield Lagrange multipliers.

The previous attempts to avoid the difficulties pointed above were not realized completely in the case of D=10 superparticle [7] but have led to the construction of pure geometrical picture of super – p – branes supersurface embedding [8] and suggestion of a generalized action principle for super – p – branes [9].

The geometrical approach to super – p – branes supported by the idea of supergravity rheonomic theory gives many advantages in comparison with known formulations. For this reason the detailed programm of investigations of extended relativistic objects in the framework of this formalism can be outlined.

The first step in this direction was made by Igor Bandos, Dmitrij Sorokin and Dmitrij Volkov [9] where a generalized action principle for super – p – branes was proposed.

One of the next steps of this approach [10], applied to the D=4 supermembranes is shortly presented in this report.

Let us start from the following action for D=4 closed supermembrane [9]:

$$S_{D=4,2} = -\frac{1}{2}\int_{\mathcal{M}_3}\left(E^{a_0}e^{a_1}e^{a_2}\varepsilon_{a_0a_1a_2} - \frac{2}{3}e^{a_0}e^{a_1}e^{a_2}\varepsilon_{a_0a_1a_2}\right) \pm \frac{4}{3}\int_{\mathcal{M}_3}\Pi^{\underline{m_2}}\Pi^{\underline{m_1}}\Theta\Gamma_{\underline{m}_1\underline{m}_2}d\Theta, \tag{2}$$

The main features of this action is that it is constructed out of differential forms being target and world supersurface vielbeins without any application of Hodge operation and is integrated over the *bosonic submanifold* $\mathcal{M}_3$ of the whole world supersurface; $\varepsilon_{a_0a_1a_2}$ is the unit antisymmetric tensor and membrane tension is chosen to be one.

So, for the construction of this action we choose a basis of one forms on a world supersurface

$$e^A = (e^a, e^\alpha) \quad ; \quad e^A = e^A{}_M dz^M \tag{3}$$

and the latter is parameterized by the coordinates $z^M = \{\xi^a, \eta^\alpha\}$.

An arbitrary local frame in the flat target superspace can be obtained from the pullback onto world supersurface of the basic supercovariant forms [5]

$$\Pi^{\underline{m}} = dX^{\underline{m}} - id\Theta\sigma^{\underline{m}}\bar{\Theta} + i\Theta\sigma^{\underline{m}}d\bar{\Theta}, \qquad d\Theta^{\underline{\mu}}, \qquad d\bar{\Theta}^{\underline{\dot{\mu}}} \tag{4}$$

by $SO(1, D-1)$ rotations

$$E^{\underline{a}} = \Pi^{\underline{m}} u_{\underline{m}}^{\underline{a}}, \quad E^{\underline{\alpha}} = d\Theta^{\underline{\mu}} \vartheta_{\underline{\mu}}^{\underline{\alpha}}, \quad \bar{E}^{\underline{\dot{\alpha}}} = d\bar{\Theta}^{\underline{\dot{\mu}}} \bar{\vartheta}_{\underline{\dot{\mu}}}^{\underline{\dot{\alpha}}} \tag{5}$$

with the vector $u_{\underline{m}}^{\underline{a}}$ and spinor $(\vartheta_{\underline{\mu}}^{\underline{\alpha}}, \bar{\vartheta}_{\underline{\dot{\mu}}}^{\underline{\dot{\alpha}}})$ components of the local Lorentz frame (supervielbein) in the target superspace, belonging to the $SO(1, D-1)$ and doubly covering for the latter $Spin(1, D-1)$ groups respectively [4] (our index notations and conventions are closely related to the ones in ref. [9]).

The superfield equations of motion derived by variation over $u_{\underline{m}}^{a}$ and e^a variables

$$\Pi^{\underline{m}} u_{\underline{m}}^{\perp} e^{a_1} e^{a_2} \varepsilon_{a_0 a_1 a_2} = 0$$

$$(\Pi^{\underline{m}} u_{\underline{m}}^{a_0} - e^{a_0}) e^{a_1} \varepsilon_{a_0 a_1 a_2} = 0 \tag{6}$$

lead to a part of the rheotropic conditions

$$E^a \equiv \Pi^{\underline{m}} u_{\underline{m}}^{a} = e^a \tag{7}$$

which relate the vector target space vielbeins to the world supersurface ones and being the standard relations of geometrical approach [11] which can be put by hands. In the framework of our consideration these relations are obtained from the variational principle and mean that e^a is induced by embedding.

Having in mind the rheotropic conditions (7) and expression for pullback vector one – form

$$\Pi^{\underline{m}} = e^\alpha \Pi_\alpha^{\underline{m}} + e^a \Pi_a^{\underline{m}} \tag{8}$$

we obtain the "geometrodynamical" condition [6] which was used previously as one specifying embedding world supersurface into a target space

$$\Pi_\alpha^{\underline{m}} = D_\alpha X^{\underline{m}} - iD_\alpha\Theta\sigma^{\underline{m}}\bar{\Theta} + i\Theta\sigma^{\underline{m}} D_\alpha\bar{\Theta} = 0 \tag{9}$$

The integrability condition for (9)

$$\gamma^a{}_{\alpha\beta} \Pi_a^{\underline{m}} = D_\alpha\Theta\sigma^{\underline{m}} D_\beta\bar{\Theta} \tag{10}$$

looks like vector – spinor relations

$$u_a^{\underline{\alpha\dot{\alpha}}} = (\gamma_a)^{\alpha\beta} \vartheta_\alpha^{\underline{\alpha}} \bar{\vartheta}_\beta^{\underline{\dot{\alpha}}}$$

and we can restrict some components of target space basic one – forms to the following values [1]

$$\Pi_a^{\underline{m}} \sim u_a^{\underline{m}}; \qquad D_\alpha \Theta^{\underline{\beta}} \sim \vartheta_\alpha^{\underline{\beta}} \tag{11}$$

After such a choice the appearence of Virasoro – like constraint

$$\Pi_a^{\underline{m}} \Pi_{\underline{m}b} = \eta_{ab} \tag{12}$$

in addition to the "geometrodynamical" condition (9) becomes perfectly clear.

But on the other hand the integrability condition for (8)

$$d\Pi^{\underline{m}} = -2id\Theta\sigma^{\underline{m}} d\bar{\Theta} = T^a u_a^{\underline{m}} + e^a D u_a^{\underline{m}} \tag{13}$$

involves the world supersurface torsion

$$T^a = De^a \equiv de^a - \Omega^a{}_b e^b \tag{14}$$

with the $\Omega^{ab}(d) = \frac{1}{2} u_{\underline{m}}^a du^{\underline{m}b}$ being the $SO(1,2)$ connection induced by the embedding, i.e. [8]

$$\Omega^{ab}(D) = 0 \tag{15}$$

The requirement (11) demands some restrictions on the world supersurface torsion. In particular [2]

$$T^a = -2id\Theta\sigma^{\underline{m}} d\bar{\Theta} u_{\underline{m}}^a \tag{16}$$

and, consequently,

$$T_{\alpha\beta}^a = -2i(\gamma^a)_{\alpha\beta}$$

$$T_{\alpha b}^a = 0; \qquad T_{cb}^a = -i\chi_c \gamma^a \chi_b \tag{17}$$

where $2Im\chi_c^\alpha \equiv D_c \Theta^{\underline{\alpha}} \vartheta_{\underline{\alpha}}^\alpha - D_c \bar{\Theta}^{\underline{\dot{\alpha}}} \bar{\vartheta}_{\underline{\dot{\alpha}}}^\alpha$ is the matter superfield defined below [3].

Equations of motion derived from the action (2) by the variation over $\Theta^{\underline{\alpha}}$ and $\bar{\Theta}^{\underline{\dot{\alpha}}}$ variables have the form of:

$$d\bar{\Theta}^{\underline{\dot{\alpha}}} u_{\underline{\alpha\dot{\alpha}}}^{a_0} e^{a_1} e^{a_2} \varepsilon_{a_0 a_1 a_2} \pm \varepsilon_{\underline{\dot{\alpha}\dot{\beta}}} \Pi^{\underline{\dot{\alpha}}}{}_{\underline{\alpha}} \Pi^{\underline{\dot{\beta}\beta}} d\Theta_{\underline{\beta}} = 0 \tag{18}$$

$$+h.c.$$

Analysis of (18) with taking into account a part of the rheotropic conditions (7) and an analog of Weyl symmetry

$$e^a \to \hat{e}^a = W^2 e^a$$

$$e^\alpha \to \hat{e}^\alpha = We^\alpha - \frac{1}{2} ie^b (\gamma_b)^{\alpha\beta} D_\gamma W \tag{19}$$

[1] To prove the rought equality we need to take into account an analog of Weyl symmetry (19) and fix an appropriate gauge.

[2] This result completely coincides with integrability condition for the rheotropic relations (7).

[3] First constraint of (17) is a backbone for all the SYM and supergravity superfield formulations giving a possibility to extract the physical sector of the theory.

leads to the remaining part of the rheotrotic relations (we fix the gauge $W = -1$)

$$E^{\alpha} \equiv d\Theta^{\underline{\alpha}} \vartheta^{\alpha}_{\underline{\alpha}} = e^{\alpha} + i e^{a} \chi^{\alpha}_{a}$$

$$\bar{E}^{\alpha} \equiv d\bar{\Theta}^{\underline{\dot{\alpha}}} \bar{\vartheta}^{\alpha}_{\underline{\dot{\alpha}}} = e^{\alpha} - i e^{a} \chi^{\alpha}_{a} \tag{20}$$

together with supermembrane equation of motion:

$$(\gamma^{a})_{\alpha}{}^{\beta} \chi^{\alpha}_{a} = 0 \tag{21}$$

The integrability conditions for spinor part of target superspace vielbeins

$$dd\Theta^{\underline{\alpha}} = 0; \qquad dd\bar{\Theta}^{\underline{\dot{\alpha}}} = 0 \tag{22}$$

which are coincided with integrability conditions for (20), give another world supersurface torsion constraints:

$$T^{\beta} = -\frac{i}{2} e^{a} \chi^{\alpha}_{a} (\gamma_{c})_{\alpha}{}^{\beta} \Omega^{c\perp} \Rightarrow T^{\beta}_{\alpha\gamma} = 0 \tag{23}$$

expression for $SO(1,3)/SO(1,2)$ vielbein form $\Omega^{\perp}_{\alpha\beta} \sim (\gamma^{a})_{\alpha\beta} \Omega^{\perp}_{a}$ [8] in terms of χ^{γ}_{a}

$$\Omega^{\perp}_{\alpha\beta} = -2i e^{\gamma} \chi_{\underline{\alpha\beta\gamma}} - e^{b} D_{\{\alpha|} \chi_{b|\beta\}} \tag{24}$$

and restriction on the χ^{γ}_{a} superfield:

$$D_{[c} \chi^{\beta}_{a]} = 2i (\chi_{a} \gamma^{s} \chi_{c}) \chi^{\beta}_{s} \tag{25}$$

Here $\underline{\alpha\beta\gamma}$, $\{\ldots\}$ and $[\ldots]$ denote the complete symmetrization, symmetrization and antisymmetrization respectively.

Moreover, the integrability conditions (22) lead to the equation of motion (21) remaining only spin 3/2 non – trivial part of χ^{α}_{a}

$$\chi_{\alpha\beta\gamma} \equiv \gamma^{a}_{\alpha\beta} \chi_{a\gamma} = \chi_{\underline{\alpha\beta\gamma}} \tag{26}$$

Geometry of world supersurface is completely described in terms of $SO(1,2)$ connection $\Omega^{ab}(d)$ and $SO(1,3)/SO(1,2)$ vielbein $\Omega^{a\perp} = \frac{1}{2} u^{a}_{\underline{m}} du^{\underline{m}\perp}$ [11] up to rotation and translation of world supersurface at whole.

By definition Ω^{ab} and $\Omega^{a\perp}$ satisfy Maurer – Cartan equations in the form of Peterson – Codazzi equation

$$d\Omega^{a\perp}(d) - \Omega^{a}{}_{b}(d) \Omega^{b\perp}(d) = 0 \tag{27}$$

and Gauss equation

$$R^{ab}(d,d) = d\Omega^{ab}(d) - \Omega^{a}{}_{c}(d) \Omega^{cb}(d) = \Omega^{a\perp}(d) \Omega^{b\perp}(d) \tag{28}$$

Intrinsic and induced geometry of world supersurface defined by the $SO(1,2)$ connection, $SO(1,3)/SO(1,2)$ vielbein and pullback of a target space basic one – forms respectively is coincided due to expression (15) [8].

The independent part of Peterson – Codazzi equation gives another restrictions on the χ_a^α superfield and Gauss equation allows to us to get an expression for supersurface curvature $R_{\alpha\beta} \sim (\gamma^{ab})_{\alpha\beta} R_{ab}$ in terms of Rarita – Schwinger – like superfield:

$$R_{\alpha\beta} = -4e^\gamma e^\delta \varepsilon^{\varepsilon\xi} \chi_{\underline{\alpha\gamma\xi}} \chi_{\underline{\delta\varepsilon\beta}} + 4ie^b e^\delta D^\gamma \chi_{b\{\alpha|} \chi_{\underline{\gamma\delta\beta}\}} + e^b e^c D_\alpha \chi_b^\gamma D_\gamma \chi_{c\beta} \tag{29}$$

In the framework of our approach world supersurface supergravity Bianchi identities

$$DT^A = \frac{1}{2} e^B R_B{}^A \tag{30}$$

are fulfilled automatically and equations of motion for X variables are dependent from the other ones.

Thus we have a complete description of our superfield membrane formulation in terms of world supersurface supergravity and Rarita – Schwinger – like matter superfield.

Unfortunately, our formulation does not possess "off – shell" invariance: the rheotropic relations lead to the equations of motion and, consequently, there is no any possibility to construct an alternative superfield formulation a lá Galperin and Sokatchev [6] as it can be done in the cases of D=3,10 heterotic superstring.

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